Efficient Horizontal Mergers:

The Effects of Internal Capital Reallocation and Organizational Form

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Abstract

This paper analyzes the resource reallocation problem in merged firms when the merged firms have a choice to retain competition between the merging partners. We consider horizontal mergers between firms that differ only in their initial capital levels. When initial capital levels are unequal, evenly distributing total capital between merging partners improves the overall cost efficiency of the merged firm. In this case, the merger often results in the multidivisional structure (the M-form) rather than the conventional structure of completely integrating merging partners because, under the M-form, the profits from aggressive production expansion backed by efficiency-improving capital reallocation is higher than the profits from output contraction and rationalization under complete integration. Such a merger enhances market competition. On the contrary, mergers between identical firms always induce complete integration.

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I. Introduction

Salant et al. (1983) have argued that most anti-competitive mergers would not even be proposed because they are, in general, unprofitable. This controversial argument has prompted many studies on what determines the profitability of horizontal mergers. Perry and Porter (1985) point out that, assuming constant marginal cost technology, Salant et al. confine the joint production possibilities of the merged firm to a range in which the merged firm does not differ from other non-merged firms. Considering mergers under convex cost technology, Perry and Porter show that horizontal mergers can be profitable, as the merged firm becomes larger by combining the production facilities of the merging partners.

In this paper we consider the possibility of reallocating capital across merging partners along with the option of retaining intra-firm competition between them. We show that evenly distributing capital across the partners can improve profits when the merging partners remain in competition. For this reason, a merged firm may not wish to completely integrate the partners. Often, a merger induces an organizational form that allows the partners to continue to compete because of the opportunity to improve cost efficiency from capital reallocation, which is otherwise unavailable.

We consider an exogenous horizontal merger between firms competing á la Cournot. [1] Firms have access to the same cost technology and differ only in their initial capital stock. Their marginal cost is a strictly convex function of capital. A merger between two firms brings the total capital of the two firms under a single authority, the headquarters, which seeks to optimally redistribute the capital to its constituent firms. For any given capital reallocation, whether the constituent firms cooperate or compete against each other affects the total output level and the profits of the merged firm. Thus, the headquarters chooses the organizational structure for the

[1] Throughout this paper, we take the merger decisions as exogenously given to focus on the effect of capital reallocation on the profitability of horizontal mergers. Although the model can be easily extended to the case of endogenous merger, the focus of the paper would then shift to the choice of merging partners and the size of profitable mergers. For this reason, we do not discuss endogenous merger in this paper.

merging partners in accordance with how it plans to reallocate the resources between them. Two organizational structures are considered: complete integration and the M-form. Under complete integration, the merging partners completely cooperate to maximize joint profits. From resource allocation to production, all decisions are centralized at the headquarters. In contrast, the M-form, introduced by Williamson (1975), is characterized by decentralized operating decisions, while the headquarters maintains strong central control over resource allocation. The merging partners become independently operating competing divisions. Divisions are charged with maximizing "division" profits, rather than the profits of the merged firm as a whole.

We find that mergers between firms with unequal levels of initial capital are likely to choose the M-form. Under the M-form, the merged firm improves cost efficiency by symmetrically reallocating capital across its constituent firms. This is because, when the marginal cost function is strictly convex in capital, one unit of capital transfer from a larger division to a smaller division increases the efficiency of the smaller division by more than it reduces the efficiency of the larger division. Further, as the disparity in cost efficiency between the divisions decreases, the divisions become more competitive with respect to each other, which leads to an expansion of output of the merged firm. Such an expansion strategy shifts rents from the rivals. Thus, the merged firm profits under the M-form due to expanded production and efficiency-enhancing capital reallocation. Had the merged firm chosen complete integration, contraction of output would have occurred owing to reduced market competition. The merged firm optimally chooses the M-form because the increase in profit from aggressive output expansion is higher than the increase in profit from contracting output under complete integration. Such a merger is also pro-competitive—the merger decreases market price.

In contrast, for a merger between firms with equal levels of initial capital, it is always optimal to induce complete integration. As the merging partners are identical in their initial capital stocks, they are already at the efficient point of capital distribution. Thus, the M-form can only generate, at most, the same profits that the merging partners were earning before the merger. On the other hand, under complete integration, the merged firm may increase profits by contracting and rationalizing outputs (Perry and Porter (1985)). Under complete integration, capital reallocation has no impact on profits. This is because equivalent results can be achieved by rationalizing output across facilities under the central control of the headquarters.

The capital reallocation strategy, output expansion strategy, and organizational form of the merged firm are complementary. The merged firm prefers to retain intra-firm competition only when output expansion is more profitable, and in that case, output expansion is induced by capital reallocation that reduces the disparity in cost efficiency between the merging partners. In contrast, the firm prefers complete integration only when output contraction is more profitable, and in that case, output contraction is induced by capital reallocation that eliminates internal competition between the merging partners.

The idea of reshuffling capital across firms has been considered by Salant and Shaffer (1998, 1999) in the context of R&D investment. They consider a Cournot oligopoly model where firms' R&D investment decisions affect their marginal cost. They show that, for a predetermined level of total investment by the members of a joint venture, an exogenous policy that induces unequal shares of the investment between the members enlarges the overall profits of the venture. In contrast, in this paper we find that equal distribution of capital across firms can be optimal. In Salant and Shaffer's framework, each firm produces at constant marginal cost and that marginal cost is decreasing in capital linearly. In this paper, the marginal cost function is strictly convex in capital. We show that the results in Salant and Shaffer do not extend to the case where the marginal cost function is strictly convex in capital.

This paper proceeds as follows. Section II briefly discusses the relevance of the M-form in contemporary corporate structures. Section III describes the model and derives the optimal solution to the merged firm's capital reallocation problem. The last section summarizes the major findings in the paper.

II. Decentralized Production and Resource Allocation

In many contemporary mergers, firms choose to keep some degree of competition alive between the merging partners after the merger. After Volvo and Ford merged, Ford's luxury brand, Jaguar, continued to compete with Volvo. Similar pattern was observed when Daimler and Chrysler merged. When Kimberly Clark and Scott Paper merged, Kleenex, the leading brand of Kimberly Clark in the facial tissue market, remained in competition with Scottie, the leading brand of Scott Paper. Had these mergers resulted in complete integration of the merging partners as is assumed in most merger theories, the competition between merging partners should have disappeared after the merger.

Jacquemin et al. (1989) report that mergers and takeovers in 1980s rarely resulted in "fusion," in which two companies genuinely become one. Many studies (See, e.g., Lang and Stulz (1994) and Fauli-Oller and Giralt (1995)) address the efficiency of decentralized organizational structures. [2]

Firms' strategic incentives for decentralized production have been discussed in many recent studies. Baye et al. (1996) model firms' strategic incentives to divide production among autonomous competing units through divisionalization, franchising, or divestiture, and derive equilibrium numbers of competing units. Tan and Yuan (2003) propose that product-line complementarities motivates divestiture of competing conglomerates. They show that, if the firms are able to coordinate their divestiture strategies, monopoly prices and profits can be achieved via a non-cooperative pricing game.

Several papers have examined the role of organizational structure in mergers. Most of them focus on how managerial compensation schemes under the delegation structure affect the

[2] Fligstein (1985) examines the spread of the M-form among the 100 largest non-financial corporations from 1919 to 1979, and reports that the proportion of the M-form corporations rose to 84.2% by 1979, from 1.5% in 1929. In his study of organizational structure in fifty of the largest companies in the United States, Chandler (1962) concludes that companies driven by market growth and technological change to diversify their products and markets could manage their new strategies efficiently only if they adopted the M-form. The reason this structural form proved so powerful was because it defined a new set of management roles and relationships that emphasized decentralization, controlled by strong corporate management, that also made the company's entrepreneurial decisions about *resource allocation*.

profitability of mergers. Ziss (2001) and Gonzalez-Maestre and Lopez-Cunat (2001) examine incentive contracts between agents and owners in the presence of the agency problem. They show that, without efficiency gains, the minimum market share that the merging parties require in order to merge profitably is substantially smaller with delegation than without it. Creane and Davidson (2004) consider a case where a merged firm operates under a multidivisional structure in which some divisions play Stackelberg leaders and the others become followers.

In this paper, the merged firm has a choice of whether to completely integrate the merging partners. We investigate when the merged firm prefers to keep internal competition between merging partners. Several papers have proposed explanations as to why merged firms might prefer to retain internal competition between the merging partners. Kamien and Zang (1990) propose that an acquiring firm may want to keep acquired rivals as competing divisions in order to deter rent shifting to other firms. Tombak (2002) considers a model of sequential mergers where the acquirer decides whether to consolidate the acquired firms. In his model, mergers always improve cost efficiency, as the acquirer transfers technology to newly acquired firms. This study shows that the acquirer has an incentive not to consolidate the acquired firm and to keep the intra-firm competition between the firms, especially when the acquirer has interests in subsequent acquisitions, in order to keep the potential target firms small and to reduce business stealing by rivals. In mergers between two hierarchical firms in a differentiated market, Ziss (2007) analyzes how the corporate center organizes incentive competition schemes for product division managers and output (or price) competition among local units. He finds that, under output competition, if the market is concentrated, a profitable merger strategy induces output contraction and complete cooperation between product division managers and local units, whereas output expansion and retaining incentive competition at the product division managers is more profitable if the market is not concentrated.

This paper shows that the merged firm may prefer to keep internal competition between the merging partners because, under such a structure, the overall cost efficiency of the firm can be improved by symmetrically reallocating capital across the partners, and with the improved cost efficiency, the merged firm expands production and enhances profits.

III. The Model

Consider an industry comprised of N initially active firms competing á la Cournot. For simplicity, we assume a linear market demand given by:

$$P = a - bQ \tag{1}$$

All firms have access to the same cost technology C(·). With the total cost of firm i equal to $C(q_i, K_i) = \frac{q_i^2}{2K_i}$,[3] the marginal cost $\frac{q_i}{K_i}$ is a decreasing function of each firm's capital stock K_i. This cost function is the dual of the Cobb-Douglas production function $q = A\sqrt{LK}$ and is homogeneous of degree one in capital and output.

In the Cournot competition prior to merger, each firm i produces at the level of output that maximizes profits, $\pi_i = (a - bQ)q_i - \frac{q_i^2}{2K_i}$. Firm i's first-order condition, $\frac{\partial \pi_i}{\partial q_i} = 0$, is

$$q_i = \frac{\beta_i}{b}(a - bQ), \qquad i = 1,2, ...N, \tag{2}$$

where $\beta_i \equiv \frac{bK_i}{bK_i + 1}$ for a given K_i, $i=1,2,....N$. Firms can differ in their initial levels of capital K_i.[4] From (2), β_i determines firm i's output responsiveness to any changes in price. As β_i increases, firm i can credibly expands its output against competitors. Thus, we interpret that β_i measures how aggressive firm i can be in production. We assume that the demand curve intersects firms' marginal cost curve from above, i.e.; that

$$\frac{\partial^2 C(q_i, K_i)}{\partial q_i^2} > |P'(Q)| \quad \Leftrightarrow \quad 1 > bK_i \quad \Rightarrow \quad \beta_i = \frac{bK_i}{bK_i + 1} < \frac{1}{2}. \tag{3}$$

The industry output is

[3] This is the cost function used by McAfee and Williams (1992) and Perry and Porter (1985). This specific framework of linear demand and quadratic cost functions is not essential in determining our main result that merged firms can improve their profits by reallocating their internal resources and optimizing their organizational structure.

[4] Sequential entry and imperfect capital markets might explain why firms evolve asymmetrically.

$$Q^* = \frac{a}{b}\left(\frac{\beta_1+\beta_2+\cdots+\beta_N}{\beta_1+\beta_2+\cdots+\beta_N+1}\right) = \frac{a}{b}\left(\frac{B}{B+1}\right), \tag{4}$$

where $B \equiv \sum_{i=1}^{N} \beta_i$. Thus, prior to merger, each firm *i* produces $q_i^* = \frac{a}{b}\frac{\beta_i}{(B+1)}$ and earns

$$\pi_i^* = \frac{a^2}{2b}\left(\frac{\beta_i+\beta_i^2}{(\beta_i+\beta_{-i}+1)^2}\right), \quad where \quad \beta_{-i} \equiv \sum_{j\neq i}\beta_j \text{ , } i = 1,2, \ldots N. \tag{5}$$

Now consider a merger between two firms, firm 1 and firm 2, whom we call "insiders." The merger brings the capital of firm 1, K_1, and that of firm 2, K_2, under the control of the headquarters of the merged firm. Assume that $K_1 \leq K_2$. The capital is fixed in the sense that it cannot easily be destroyed or increased in a short period. However, we assume that the capital is mobile in that it can easily be reallocated from one operating unit to another.[5]

The merged firm's optimization problem consists of two stages. In the first stage, the headquarters optimally reallocates the corporate resources $K_M = K_1 + K_2$ to the insiders. At the end of this stage, how the resources have been reallocated determines whether the firm will operate under the M-form, which completes the restructuring process after the merger. In the second stage, the merged and non-merging firms resume competition and simultaneously determine their output levels. If the corporate structure of the merged firm is the M-form, each division *j* competitively chooses its output level for a given level of capital. If the insiders completely integrate, the headquarters also centrally controls output decisions.

3.1 Production Stage under the M-form

Suppose that the merged firm is operating under the M-form. Let K_{Mj} be the level of capital that each division *j* has received from the headquarters in the first stage. Since the two divisions compete with one another, each division's best response function is the same as before the merger, as shown in equation (2), except that the new level of capital K_{Mj} that it receives from

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[5] Human capital, know-how, brands, shelf space, etc. would fit into the definition of this type of capital. In reality, this type of medium-run, fixed but mobile capital accounts for substantial portion, but not all, of total capital for firms. For simplicity, in this model, we assume that the entire capital of the insiders is mobile. Limiting the fraction of mobile capital does not alter the main result that merged firms' profits can improve as a result of internal capital reallocation, except that it reduces the range of profitable capital reallocation.

the headquarters may differ from its pre-merger level of capital K_j, $j=1,2$. In the second stage, for a given K_{Mj}, division j's best response is

$$q_{Mj} = \tfrac{\beta_{Mj}}{b}(a - bQ) \quad j = division\, 1, 2\,, \tag{6}$$

where $\beta_{Mj} \equiv \frac{bK_{Mj}}{bK_{Mj}+1}$, for $j = 1, 2$. The best response of rival firm l is given by

$$q_l = \tfrac{\beta_l}{b}(a - bQ), \quad l = 3, 4, \ldots, N\,, \tag{7}$$

where $\beta_l \equiv \frac{bK_l}{bK_l+1}$, $\quad l = 3, 4, \ldots., N$. The industry output and price depend on β_M and β_{-M}.

$$Q_M = \frac{a(\beta_{M1} + \beta_{M2} + \beta_3 \cdots + \beta_N)}{b(\beta_{M1} + \beta_{M2} + \beta_3 \cdots + \beta_N + 1)} = \frac{a}{b}\left(\frac{\beta_M + \beta_{-M}}{\beta_M + \beta_{-M} + 1}\right), \quad \text{and}$$

$$P_M = \left(\frac{a}{\beta_M + \beta_{-M} + 1}\right). \tag{8}$$

Then, in equilibrium,

$$q_M = q_{M1} + q_{M2} = \left(\frac{\beta_M}{b}\right)\left(\frac{a}{\beta_M + \beta_{-M} + 1}\right) \text{ and } q_l = \frac{\beta_l}{b}\left(\frac{a}{\beta_M + \beta_{-M} + 1}\right). \tag{9}$$

The merged firm's profits are the joint profits of the insiders,

$$\pi_M = \pi_{M1} + \pi_{M2} = (a - bQ)q_{M1} - \frac{q_{M1}^{\ 2}}{2K_{M1}} + (a - bQ)q_{M2} - \frac{q_{M2}^{\ 2}}{2K_{M2}}. \tag{10}$$

Plugging (8) and (9) into the merged firm's profit function (10), we get

$$\pi_M = \frac{a^2}{2b}\left(\frac{\beta_{M1} + \beta_{M2} + \beta_{M1}^2 + \beta_{M2}^2}{(B_M + 1)^2}\right). \tag{11}$$

3.2 Production Stage under Complete Integration

Under complete integration, the merging partners cooperate to maximize joint profits, π_{CI}, and the headquarters rationalizes production at each facility. In the second stage, for a given $K_M = K_1 + K_2$, the merged firm's best response is

$$q_{CI} = \tfrac{\beta_{CI}}{b}(a - bQ)\,, \tag{12}$$

where $\beta_{CI} \equiv \frac{bK_M}{bK_M+1}$. The industry output is given by

$$Q_M = \frac{a(\beta_{CI} + \beta_3 \cdots + \beta_N)}{b(\beta_{CI} + \beta_3 \cdots + \beta_N + 1)} = \frac{a}{b}\left(\frac{\beta_{CI} + \beta_{-M}}{\beta_{CI} + \beta_{-M} + 1}\right). \quad (13)$$

In equilibrium, the output levels of the merged firm and a non-merging firm *l*, respectively, are

$$q_{CI} = \frac{\beta_{CI}}{b}\left(\frac{a}{\beta_{CI} + \beta_{-M} + 1}\right) \text{ and } q_l = \frac{\beta_l}{b}\left(\frac{a}{\beta_{CI} + \beta_{-M} + 1}\right). \quad (14)$$

The merged firm's profits are

$$\pi_{CI} = (a - bQ)q_{CI} - \frac{{q_{CI}}^2}{2K_M} = \frac{a^2}{2b}\left(\frac{\beta_{CI} + \beta_{CI}^2}{(\beta_{CI} + \beta_{-M} + 1)^2}\right). \quad (15)$$

3.3 Capital Reallocation under the M-form

In the first stage, the headquarters seeks to optimally determine the distribution of capital for the insiders under the constraint that $K_M = K_1 + K_2$. Consider the following distribution rule ε for the two divisions under the M-form:

$$(K_{M1} = K_1 + \varepsilon, \quad K_{M2} = K_2 - \varepsilon), \quad (16)$$

where $-K_1 < \varepsilon \le \frac{K_2 - K_1}{2}$. [6] Let $\varepsilon_S = \frac{K_2 - K_1}{2}$ denote the capital reallocation that makes the two divisions symmetric and identical, which we call "symmetric reallocation." At $\varepsilon_S = \frac{K_2 - K_1}{2}$, $K_{Mj} = K_S = \frac{K_M}{2}$, *j*=1,2. The range includes a capital transfer from the first division to the second division, ε ≤ 0, which increases the concentration in this industry.

A small capital transfer from division 2 to division 1 improves β_{M1} and π_{M1} and reduces β_{M2} and π_{M2}. Nonetheless, both divisions continue to compete and produce non-zero outputs under the M-form. However, if one division gets all of the capital, $\varepsilon = -K_1$, the other division can no longer produce. Such a capital reallocation eliminates the internal competition between the two divisions. Thus, at $\varepsilon = -K_1$, the capital reallocation completely integrates production. Conversely, if the optimal reallocation occurs in the range $-K_1 < \varepsilon \le \frac{K_2 - K_1}{2}$, the resulting corporate

[6] In the range $\frac{K_2 - K_1}{2} < \varepsilon < K_2$, the capital transfer makes the second division smaller than the first division. Since the dual outcome of such a capital transfer can be found in the range $-K_1 < \varepsilon \le \frac{K_2 - K_1}{2}$, the relevant range for a capital transfer is $-K_1 < \varepsilon \le \frac{K_2 - K_1}{2}$.

structure is the M-form. That is, if both divisions receive a non-zero level of capital at the optimal reallocation, the optimal internal structure of the merged firm is the M-form, and, if the optimal reallocation under the M-form does not have an interior solution, the optimal internal structure is complete integration. In this way, the optimal capital reallocation and organizational structure of the merged firm are jointly determined by the process of capital reallocation.

Let us examine how the merged firm allocates capital if it operates under the M-form. In Lemma 1, we derive the condition under which increasing symmetry in the capital distribution of divisions increases joint profits. In Proposition 1, we prove that, if the merger operates under the M-form, the merged firm maximizes profit by reallocating capital symmetrically to the insiders, i.e., at $\varepsilon_S = \frac{K_2 - K_1}{2}$.

LEMMA 1: *Consider the problem of distributing a fixed size of capital across a pair of firms, firm 1 and firm 2, in an industry with linear demand and quadratic cost functions. Increasing symmetry in the capital distribution of the two firms improves joint profits if and only if*

$$\beta_S > \frac{(1-3\beta_{-M})}{2}, \qquad (17)$$

where β_S *is each firm's output expansion parameter at the symmetric reallocation* $\varepsilon_S = \frac{K_2 - K_1}{2}$.

PROOF: See the Appendix.

PROPOSITION 1: *Consider a merger between two firms, firm 1 and firm 2, when* $K_1 \leq K_2$ *and* $K_1 + K_2 = K_M$*. If the merged firm operates under the M-form, the unique solution to the optimal capital reallocation problem is to move* $\varepsilon_S = \frac{K_2 - K_1}{2}$ *from division 2 to division 1.*

PROOF: See the Appendix.

From Lemma 1, increasing symmetry in the capital distribution across the insiders is profitable if the condition in (17) is satisfied. Hence, under this condition, the merged firm maximizes profit by redistributing K_M evenly between the insiders. If this condition is not met, however, the merged firm improves profit by increasing asymmetry in capital distribution as much as possible. In this case, there is no interior solution to the capital reallocation problem that maintains the M-form because a division without capital cannot produce. Therefore, if a merged firm chose to operate under the M-form, it must be the case that increasing symmetry in capital distribution improves profits. Then, under the M-form, the optimal capital reallocation is to distribute the capital symmetrically across the insiders.

Reducing asymmetry in the capital distribution across the insiders improves the merged firm's profit because it improves the cost efficiency of the merged firm. Prior to merger, the merging partners' marginal costs are $\frac{q_1}{K_1} = \frac{a(1-\beta_1)}{(1+B)}$ and $\frac{q_2}{K_2} = \frac{a(1-\beta_2)}{(1+B)}$. Let $mc_M(q_1, q_2, K_1, K_2) \equiv \frac{q_1}{K_1} + \frac{q_2}{K_2}$ be the sum of marginal costs of the insiders before capital reallocation. Then, for $K_1 < K_2$, an infinitesimal amount of capital transfer $\varepsilon > 0$ from division 2 to division 1 enhances the overall cost efficiency of the merged firm:

$$\left.\frac{dmc_M}{d\varepsilon}\right|_{\varepsilon\to 0} = \left(\frac{\partial(q_1/K_1+\varepsilon)}{\partial\varepsilon} + \frac{\partial(q_1/K_2-\varepsilon)}{\partial\varepsilon}\right) = \frac{a}{(1+B)^2}\left[-\left(\frac{\partial\beta_1}{\partial K_1} - \frac{\partial\beta_2}{\partial K_2}\right)(3+B-\beta_1-\beta_2)\right] < 0 \qquad (18)$$

since $\left(\frac{\partial\beta_1}{\partial K_1} - \frac{\partial\beta_2}{\partial K_2}\right) > 0$. This is because, in the pre-merger setting, when $K_1 < K_2$, the total production costs of the two merging partners are not minimized. The most cost efficient operation of the two divisions occurs when total capital K_M is allocated evenly across the two divisions. Such an efficiency-improving capital reallocation always exists for any merger with asymmetric insiders.

On the other hand, capital reallocation also indirectly increases total production costs, because improved efficiency increases the merged firm's output. As the merged firm produces more, the firm's revenue increases for a given market price while the increased output also lowers market price. Overall, if the effect on market price and the increase in total costs are small enough, the efficiency-improving symmetric reallocation of capital improves the merged firm's profits, which is the interpretation of the condition in (17). Obviously, it depends on the shapes of the market demand and cost functions. In the current framework of linear demand and quadratic cost functions, a necessary and sufficient condition for increasing symmetry in the capital distribution of the insiders to improve the merged firm's profits reduces to $\beta_S > \frac{(1-3\beta_{-M})}{2}$, where β_S is each insider's output expansion parameter at the symmetric reallocation $\varepsilon_S = \frac{K_2 - K_1}{2}$. That is, distributing total capital equally across the insiders improves the merged firm's profits if and only if $\beta_S > \frac{(1-3\beta_{-M})}{2}$. Otherwise, the merged firm is better off by increasing asymmetry between the insiders, which leads to the breakdown of the M-form. At $\varepsilon_S = \frac{K_2 - K_1}{2}$, $\beta_M = \beta_S + \beta_S = 2\beta_S$. From (8), a large β_{-M} and a large β_S imply that capital reallocation does not reduce price much since price level is already so low. Also in this case, from (9), the merged firm's output does not increase much, which increases the total cost by only a small amount. Thus, it is likely that symmetric reallocation of capital improves the merged firm's profits.

This result contrasts with the results in Salant and Shaffer (1998, 1999). They find that, keeping the sum of marginal costs constant, increasing asymmetry in the marginal costs of identical firms increases the joint profits of the firms. They consider a model where a unit of R&D investment reduces a firm's marginal cost proportionally, i.e., $mc_i = \overline{c} - K_i$. The difference between the results of Salant and Shaffer and the result of this paper stems from the fact that Salant and Shaffer use a marginal cost function that is not strictly convex in capital while the marginal cost function in this paper is strictly convex in capital.[7] In Salant and Shaffer (1998, 1999), $\frac{\partial^2 C(q_i, K_i)}{\partial K_i^2} = 0$ and $\frac{\partial^2 mc_i}{\partial K_i^2} = 0$, which implies that one unit of capital transfer affects the marginal costs of the giver and the recipient to the same extent, and thus, there is no change in the sum of marginal costs. For such cost technology, increasing symmetry in the capital distribution of firms only increases aggregate costs (Bergstrom and Varian (1985a,b)). Thus, it is always better to increase asymmetry in capital distribution to minimize aggregate costs.

[7] Using a general cost function, we show in the Supplementary Appendix that the possibility of improving joint profits by distributing capital equally across firms arises only if the marginal cost function is strictly convex in capital.

Conversely, in this paper, $\frac{\partial^2 C(q_i, K_i)}{\partial K_i^2} > 0$ and $\frac{\partial^2 mc_i}{\partial K_i^2} > 0$. The condition $\frac{\partial^2 mc_i}{\partial K_i^2} > 0$ implies that one unit of capital transfer increases the giver's marginal cost more than it reduces the recipient's marginal cost if the giver is smaller than the recipient. Therefore, increasing asymmetry in capital distribution increases the sum of the marginal costs of the two firms. In order to keep the sum of marginal costs constant, the firms would have to raise additional capital, which is not feasible in the current framework. Rather, if the firms equalize their capital levels, overall cost efficiency improves, which can be profitable if $\beta_S > \frac{(1-3\beta_{-M})}{2}$. Hence, the results from Salant and Shaffer (1998, 1999) do not extend to the case where marginal cost is a strictly convex function of capital.

3.4 Capital Reallocation under Complete Integration

Under complete integration, capital reallocation is irrelevant. That is, whether the entire capital is monopolized by a single plant or divided among many plants makes no difference in terms of the firm's strategic output decision, if the merger operates under complete integration.

LEMMA 2: *When a merger completely integrates the production of the insiders, the merged firm's profit is invariant to any capital reallocation.*

PROOF: See the Appendix.

Lemma 2 proves that locating all production in one division is equivalent, in terms of the firm's strategic behavior, to all other types of completely integrated production in which the headquarters of a multi-plant firm centrally organizes each plant's output level. This is because, under complete integration, there is essentially only one production decision unit regardless of the number of operating units.

3.5 Optimal Organizational Form

The merged firm adopts the M-form only if it is more profitable than complete integration. If increasing symmetry in capital distribution improves profits, i.e., $\beta_S > \frac{(1-3\beta_{-M})}{2}$, the maximum profit under the M-form occurs at $\varepsilon_S = \frac{K_2 - K_1}{2}$ (Proposition 1). Since capital reallocation has no impact on profit for a fixed K_M under complete integration (Lemma 2), comparing the profit at $\varepsilon_S = \frac{K_2 - K_1}{2}$ under the M-form with the profit under complete integration, we can determine whether the M-form is optimal for the merged firm in this case. However, if increasing symmetry in capital distribution reduces profits, i.e., $0 < \beta_S < \frac{(1-3\beta_{-M})}{2}$, the merged firm allocates the entire capital to one division. In this case, the merger can profit from complete integration.

As all firms in the industry are identical except for their initial levels of capital, there can be only two types of mergers: a merger between firms with initially unequal levels of capital, which we call a "merger between non-identical firms," and a merger between firms with initially equal levels of capital, which we call a "merger between identical firms." The M-form can be optimal for mergers between non-identical firms, and complete integration is optimal for all mergers between identical firms.

3.5.1 Organizational Form for a Merger between Non-identical Firms

In the case of a merger between non-identical firms, at $\varepsilon_S = \frac{K_2 - K_1}{2}$, $\beta_{Mj} = \beta_S$. Therefore, at ε_S, the merged firm's profit π_M^* is

$$\pi_M^* = \frac{a^2}{2b}\left(\frac{\beta_{M1} + \beta_{M2} + \beta_{M1}^2 + \beta_{M2}^2}{(\beta_{M1} + \beta_{M2} + \beta_{-M} + 1)^2}\right) = \frac{a^2}{2b}\left(\frac{2\beta_S + 2\beta_S^2}{(2\beta_S + \beta_{-M} + 1)^2}\right). \quad (19)$$

From (15), under complete integration, the merged firm earns

$$\pi_{CI} = \frac{a^2}{2b}\left(\frac{\beta_{CI} + \beta_{CI}^2}{(\beta_{CI} + \beta_{-M} + 1)^2}\right).$$

Using $K_i = \frac{\beta_i}{b(1-\beta_i)}$, we can rewrite that $\varepsilon_S = \frac{K_2 - K_1}{2} = \frac{\beta_2 - \beta_1}{2b(1-\beta_1)(1-\beta_2)}$. Then, $\beta_M = 2\beta_S = \frac{2(\beta_1 + \beta_2 - 2\beta_1\beta_2)}{(2-\beta_1-\beta_2)} > \beta_1 + \beta_2$, making the merged firm more aggressive in production. In contrast, when complete integration occurs, combining the total capital makes the merged firm less aggressive in production, i.e., $\beta_{CI} \equiv \frac{b(K_1+K_2)}{b(K_1+K_2)+1} < \beta_1 + \beta_2 \equiv \frac{bK_1}{bK_1+1} + \frac{bK_2}{bK_2+1}$. In summary,

$$\beta_M = \begin{cases} 2\beta_S \ (> \beta_1 + \beta_2) & \text{under the M-form} \\ \beta_{CI} \ (< \beta_1 + \beta_2) & \text{under complete integration.} \end{cases}$$

The M-form is optimal if and only if $\pi_M^* > \pi_{CI}$ and $\pi_M^* > \pi_1 + \pi_2$.[8] Since we are considering mergers with a fixed level of total capital K_M, we get $\beta_S = \frac{b(K_M/2)}{b(K_M/2)+1} = \frac{\beta_{CI}}{2-\beta_{CI}}$. Then, from (15) and (19),

$$\pi_M^* = \frac{a^2}{2b}\left(\frac{2\beta_S + 2\beta_S^2}{(2\beta_S + \beta_{-M} + 1)^2}\right) > \frac{a^2}{2b}\left(\frac{\beta_{CI} + \beta_{CI}^2}{(\beta_{CI} + \beta_{-M} + 1)^2}\right) = \pi_{CI}. \tag{20}$$
$$\Leftrightarrow \frac{(\beta_{-M}+1)(3\beta_{-M}-1)}{(\beta_{-M}-1)^2} > \beta_{CI}$$

$\pi_M^* > \pi_{CI}$ if β_{-M} is large, β_{CI} (or β_S) is small, or both. For $\beta_{-M} > \sqrt{2}-1$, $\frac{(\beta_{-M}+1)(3\beta_{-M}-1)}{(\beta_{-M}-1)^2} > 1$, and thus, $\frac{(\beta_{-M}+1)(3\beta_{-M}-1)}{(\beta_{-M}-1)^2} > 1 > \beta_{CI}$ is always satisfied. Hence, if β_{-M} is large enough, the merged firm always chooses the M-form regardless of β_{CI} (or β_S).

PROPOSITION 2: (**Optimality of the M-form for a Merger between Non-identical Firms**) *Consider a horizontal merger between firm 1 and firm 2 when $K_1 < K_2$ and $K_1 + K_2 = K_M$. If $\frac{(\beta_{-M}+1)(3\beta_{-M}-1)}{(\beta_{-M}-1)^2} > \beta_{CI}$, the merged firm's profits are higher under the M-form than under complete integration and, thus, the merged firm optimally chooses the M-form.*

PROOF: See the Appendix.

Under the M-form, the merged firm profits from aggressive production. Symmetric capital reallocation reduces the difference in cost efficiency between the two divisions, which amplifies competition between the divisions and makes the merged firm as a whole more aggressive than before, generating output expansion. From (9),

[8] The result can easily be extended to the case of a positive merger cost $\delta > 0$. Mergers are profitable under the M-form if $\pi_M^* - \pi_{CI} > \delta$ and $\pi_M^* - (\pi_1 + \pi_2) > \delta$, which simply implies that the range of parameters where mergers are profitable is reduced when we consider $\delta > 0$. But the qualitative results of this paper would not be affected.

$$q_M = q_{M1} + q_{M2} = \frac{a(2\beta_S)}{b}\left(\frac{1}{2\beta_S + \beta_{-M} + 1}\right) > \frac{a(\beta_1+\beta_2)}{b}\left(\frac{1}{\beta_1+\beta_2+\beta_{-M}+1}\right) = q_1 + q_2,$$

since $2\beta_S > \beta_1 + \beta_2$. Expanded production of the merged firm incurs rent shifting from rivals. From (7), $q_l = \frac{\beta_l}{b}(a - bQ) = \frac{\beta_l}{b}\frac{a}{(B_M+1)}$ for l=3,4,....N, which is lower than the pre-merger level since $B_M = \beta_M + \beta_{-M} > B = \beta_1 + \beta_2 + \beta_{-M}$. Under complete integration, in contrast, the merged firm contracts output ($q_{CI} < q_1 + q_2$) as the merger decreases the degree of competition in the market. Then, the merged firm profits only if the merger substantially reduces market competition. As Perry and Porter (1985, p. 222) write, "the profits of the merged firm can exceed those of its constituent firms only if the merger results in a price rise sufficient to offset the lower output level."

If profits under the M-form are higher than they are under complete integration, gains from large decentralized production must outweigh gains from small centralized production under reduced market competition, and vice versa. From (8), for any given β_M, the effect of the merger on market price gets smaller as β_{-M} increases. This means that when β_{-M} is large, if the merged firm completely integrates the insiders' production, the price rise is not sufficient to offset the lower output level for the merged firm. Thus, the merged firm cannot gain much from reducing competition. Similarly, if β_{CI} (or β_S) is small, the size of total capital that the merged firm can combine is too small to induce a large price rise through reducing market competition. Rather, the firm can improve its profits by increasing its market share and lowering its costs under the M-form. This explains the condition in Proposition 2.

Since $\frac{(\beta_{-M}+1)(3\beta_{-M}-1)}{(\beta_{-M}-1)^2} > 0$ only if $\beta_{-M} > \frac{1}{3}$, if a merged firm prefers adopting the M-form over completely integrating the insiders, it must be the case that increasing symmetry in capital distribution enhances profits, i.e., $\beta_S > 0 > \frac{(1-3\beta_{-M})}{2}$. This implies that $\pi_M^* > \pi_1 + \pi_2$ if $\pi_M^* > \pi_{CI}$, where $\pi_1 + \pi_2$ is the sum of pre-merger profits of the merging partners. Thus, mergers are always profitable in this case, which gives the following result:

COROLLARY 1: *If* $\frac{(\beta_{-M}+1)(3\beta_{-M}-1)}{(\beta_{-M}-1)^2} > \beta_{CI}$, a *merger between non-identical firms is always profitable through divisionalization and capital reallocation.*

On the other hand, if $\beta_{CI} > \frac{(\beta_{-M}+1)(3\beta_{-M}-1)}{(\beta_{-M}-1)^2}$, then, $\pi_{CI} > \pi_M^*$. In this case, complete integration is optimal for the merged firm since $\pi_{CI} > \pi_M^* > \pi_1 + \pi_2$ if $\beta_S > \frac{(1-3\beta_{-M})}{2}$ and $\pi_{CI} > \pi_1 + \pi_2 > \pi_M^*$ if $\beta_S < \frac{(1-3\beta_{-M})}{2}$.

3.5.2 Optimality of Complete Integration for a Merger between Identical Firms

For a merger between identical firms, the M-form is never optimal. Under the M-form, the merged firm would earn at most the same profits as before by doing nothing; any capital reallocation makes the two divisions asymmetric and more inefficient. The merged firm can profit only from a price rise and reduced competition. Moreover, under complete integration, the headquarters rationalizes each plant's output, so the merged firm's profits do not vary with capital redistribution (Lemma 2). With complete integration of the insiders' production and output rationalization, the merged firm earns π_{CI}. Such a merger is profitable if $\beta_{CI} > \frac{(\beta_{-M}+1)(3\beta_{-M}-1)}{(\beta_{-M}-1)^2}$.

PROPOSITION 3: (**Optimality of Complete Integration for a Merger between Identical Firms**) *Consider a horizontal merger between two identical firms. The optimal organizational form for the merged firm is complete integration.*

PROOF: See the Appendix.

Table 1 summarizes the ranges of parameters in which profitable mergers arise, and the optimal organizational form of the merged firm in each case. The parameter range in which the M-form with symmetric capital distribution is superior to complete integration, $\frac{(\beta_{-M}+1)(3\beta_{-M}-1)}{(\beta_{-M}-1)^2} > \beta_{CI}$, and the range in which complete integration is superior to the M-form with symmetric capital distribution, $\beta_{CI} > \frac{(\beta_{-M}+1)(3\beta_{-M}-1)}{(\beta_{-M}-1)^2}$, are mutually exclusive and exhaustive.

Table 1. Profitability and Optimal Organizational Form of Mergers

	Merger between identical firms	Merger between non-identical firms
$\beta_{CI} > \frac{(\beta_{-M}+1)(3\beta_{-M}-1)}{(\beta_{-M}-1)^2}$	Case I: Profitable $\pi_{CI} > \pi_M^* = \pi_1 + \pi_2$ Complete integration (Perry and Porter (1985))	Case II: Profitable $\pi_{CI} > \pi_M^*$, $\pi_{CI} > \pi_1 + \pi_2$ Complete integration
$\frac{(\beta_{-M}+1)(3\beta_{-M}-1)}{(\beta_{-M}-1)^2} > \beta_{CI}$	Case III: Not profitable $\pi_1 + \pi_2 = \pi_M^* > \pi_{CI}$ (Salant et al. (1983))	Case IV: Profitable $\pi_M^* > \pi_{CI}$, $\pi_M^* > \pi_1 + \pi_2$ The M-form

How the merged firm chooses its organizational form depends crucially on whether output expansion is more profitable than output contraction. The merged firm chooses the M-form if and only if output expansion is more profitable than output contraction, and vice versa. Output expansion is profitable only if increasing symmetry in the capital distribution of the insiders improves the merged firm's profit, i.e., $\beta_S > \frac{(1-3\beta_{-M})}{2}$. Thus, if output expansion is profitable, the merged firm evenly distributes the total capital to the insiders under the M-form in order to maximize the level of output it can credibly commit to producing.

In the range $\frac{(\beta_{-M}+1)(3\beta_{-M}-1)}{(\beta_{-M}-1)^2} > \beta_{CI} > 0$, it is always true that $\beta_S > 0 > \frac{(1-3\beta_{-M})}{2}$ since $\frac{(\beta_{-M}+1)(3\beta_{-M}-1)}{(\beta_{-M}-1)^2} > 0$ only if $\beta_{-M} > \frac{1}{3}$. Since $\beta_S > \frac{(1-3\beta_{-M})}{2}$, $\pi_M^* \geq \pi_1 + \pi_2$. Therefore, in this range, $\pi_M^* > \pi_{CI}$ and $\pi_M^* \geq \pi_1 + \pi_2$. Then, the only case where a merger is not profitable is when the merging partners are identical. In that case, $\pi_M^* = \pi_1 + \pi_2$ and $\pi_1 + \pi_2 = \pi_M^* > \pi_{CI}$ (Case III). However, if merging partners are not identical, the merger is always profitable and the merged firm chooses the M-form, since $\pi_M^* > \pi_1 + \pi_2$ and $\pi_M^* > \pi_{CI}$ (Case IV, corollary 1).

In the range $\beta_{CI} > \frac{(\beta_{-M}+1)(3\beta_{-M}-1)}{(\beta_{-M}-1)^2}$, $\pi_{CI} > \pi_M^*$. If merging partners are identical, the merger is always profitable under complete integration because $\pi_{CI} > \pi_M^* = \pi_1 + \pi_2$ (Case I). A merger between non-identical firms is also profitable under complete integration. In the case where $\beta_S > \frac{(1-3\beta_{-M})}{2}$, output expansion under the M-form increases the merged firm's profits, and thus, $\pi_M^* \geq \pi_1 + \pi_2$. But since $\pi_{CI} > \pi_M^*$, the increase in profit under the M-form is not greater than the increase in profit under complete integration. Hence, the merged firm chooses complete integration in this case. On the other hand, if $0 < \beta_S < \frac{(1-3\beta_{-M})}{2}$, the greater the asymmetry in the capital distribution of the divisions, the higher is the merged firm's profit, implying that $\pi_{CI} > \pi_1 + \pi_2 > \pi_M^*$. The merger is profitable and chooses complete integration (Case II).

In Case II, the merged firm chooses complete integration even if the merger would have profited from output expansion under the M-form. This is simply because the increase in profit from output contraction under complete integration is larger than the increase in profit from output expansion under the M-form in this case. Under the M-form, the symmetric reallocation ε_S gives the local maximum profit π_M^* and the global maximum occurs at the corner solution $\varepsilon = -K_1$. Hence, the condition that symmetrically reallocating capital across the insiders improves profits, i.e., $\beta_S > \frac{(1-3\beta_{-M})}{2}$, is only a necessary condition for mergers to choose the M-form, but is not a sufficient condition.

From Table 1, we find that if firms optimize for organizational forms, horizontal mergers between non-identical firms are always profitable, either through capital reallocation under the M-form or through output rationalization under complete integration.

COROLLARY 2: *Any horizontal merger between non-identical firms is profitable.*

Horizontal mergers may not be anti-competitive. When a merged firm organizes its production under the M-form, the merger enhances competition.[9]

[9] Farrell and Shapiro (1990b) discuss a condition under which *exogenous* capital sales between oligopoly firms improve welfare. They show that a small capital sale from a larger firm to a smaller firm improves welfare. When

PROPOSITION 4: (Pro-competitive Horizontal Mergers) *Horizontal mergers lower market price, increase market output, and increase consumer surplus if the merged firm operates under the M-form.*

PROOF: See the Appendix.

IV. Conclusion

This paper examines how the capital reallocation problem and the choice of organizational form are intertwined in determining the profitability of horizontal mergers. In contrast to the findings in Salant and Shaffer (1998, 1999), we find that, often, profitable capital reallocation is obtained by reducing, rather than increasing, asymmetry in the capital distribution of the merging partners. The difference arises from the fact that Salant and Shaffer use a constant marginal cost that is linearly decreasing in capital: their results do not extend to the case where marginal cost is a strictly convex function of capital.

We also find that the M-form, in which intra-firm competition remains after the merger, is preferred by the merged firm if the merged firm gains the most from output expansion. Symmetrically reallocating capital across insiders makes the merged firm expand output aggressively. With improved cost efficiency from capital reallocation, this aggressive production expansion improves the merged firm's profit. On the other hand, if output contraction is more

the welfare effect alone is considered, it appears as though capital sales between firms achieve the same welfare-enhancing effect as does a merger operating under the M-form. However, mergers under the M-from are intrinsically different from the capital sales in one important aspect: incentives. As the capital level determines a firm's competitiveness in the market, capital sales between competing firms necessarily imply that the seller loses profits and the buyer gains profits as a result. Obviously, no profit-maximizing firm would be willing to be engaged in such capital sales unless the firm faces financial constraints that force such capital sales. Hence, typically, welfare-improving capital sales are difficult to enforce in the first place. Moreover, even if the capital sales occur, firms are never interested in achieving the level of capital sales that induces the most efficient capital distribution. Thus, capital sales can never be as efficient as capital redistribution after mergers. On the contrary, in the case of a merger, the merged firm can enforce the optimal capital reallocation because it improves its *overall* profits. Although one of the two divisions must incur losses in order for the optimal capital redistribution to work, the headquarters has the authority to enforce the resource reallocation as long as it improves the total profits. That is, merger is an enforcement mechanism for the welfare-improving capital redistribution (or capital sales).

profitable, the merged firm reallocates capital as asymmetrically as possible across the insiders in order to contract output, resulting in complete integration.

Confirming the results in Tombak (2002) and Kamien and Zang (1990), we show that the merged firm may prefer to retain intra-firm competition. In this paper, what motivates the merged firm to keep intra-firm competition is the profitable resource reallocation that induces more aggressive production by the merged firm. We find that mergers between non-identical firms often result in the M-form. Such mergers are not only privately profitable, but also socially efficient. In contrast, for mergers between identical firms, the only optimal organizational form is complete integration, under which capital reallocation is irrelevant. As Salant et al. (1983) predicted, such mergers are often unprofitable, and welfare reducing even if profitable.

APPENDIX

A1. PROOF OF LEMMA 1

Consider distributing a fixed size of capital K_M across a pair of firms, firm 1 and firm 2, where $K_M = K_1 + K_2$. Assume that $K_1 \leq K_2$. K_M is distributed according to the distribution rule ε in (16). Under the redistribution rule, each division's capital level can be rewritten as

$$\begin{aligned} K_{M1} &= \frac{\beta_{M1}}{b(1-\beta_{M1})} = \frac{\beta_1}{b(1-\beta_1)} + \varepsilon \Rightarrow \quad \beta_{M1} = \frac{\beta_1 + \varepsilon b(1-\beta_1)}{1+\varepsilon b(1-\beta_1)} = \frac{\beta_1 + x_1}{1+x_1} \\ K_{M2} &= \frac{\beta_{M2}}{b(1-\beta_{M2})} = \frac{\beta_2}{b(1-\beta_2)} - \varepsilon \Rightarrow \quad \beta_{M2} = \frac{\beta_2 - \varepsilon b(1-\beta_2)}{1-\varepsilon b(1-\beta_2)} = \frac{\beta_2 - x_2}{1-x_2} \end{aligned}, \qquad \text{(A-1)}$$

where $x_1 \equiv \varepsilon b(1-\beta_1), x_2 \equiv \varepsilon b(1-\beta_2)$. A capital transfer in the range $-K_1 < \varepsilon \leq \frac{K_2 - K_1}{2}$ always results in $\beta_{M2} \geq \beta_{M1}$ with equality at ε_S. Using $K_i = \frac{\beta_i}{b(1-\beta_i)}$, we can rewrite that $\varepsilon_S = \frac{K_2 - K_1}{2} = \frac{\beta_2 - \beta_1}{2b(1-\beta_1)(1-\beta_2)}$. At $\varepsilon_S = \frac{\beta_2 - \beta_1}{2b(1-\beta_1)(1-\beta_2)}$, $\beta_{M1} = \beta_{M2} = \frac{(\beta_1 + \beta_2 - 2\beta_1\beta_2)}{(2-\beta_1-\beta_2)} \equiv \beta_S$.

A small amount of capital transfer ε changes joint profits $\pi_M \equiv \pi_1 + \pi_2$ by

$$\frac{\partial \pi_M}{\partial \varepsilon} = \underbrace{\frac{\partial\{(a-bQ)(q_{M1}+q_{M2})\}}{\partial \varepsilon}}_{A:\Delta\ in\ \text{Revenue}} - \underbrace{\left\{\frac{\partial(q_{M1}^2)}{\partial \varepsilon}\left(\frac{1}{2K_{M1}}\right)+\frac{\partial(q_{M2}^2)}{\partial \varepsilon}\left(\frac{1}{2K_{M2}}\right)\right\}}_{B:\Delta\ in\ \text{cost}} - \underbrace{\left\{q_{M1}{}^2\left(\frac{\partial\left(1/(2K_{M1})\right)}{\partial \varepsilon}\right)+q_{M2}{}^2\left(\frac{\partial\left(\left(1/(2K_{M2})\right)\right)}{\partial \varepsilon}\right)\right\}}_{C:\Delta\ in\ \text{cost}} \cdot \quad \text{(A-2)}$$

The first term is the effect of capital reallocation on the revenue:

$$A: \ \frac{\partial\{(a-bQ)(q_{M1}+q_{M2})\}}{\partial \varepsilon} = \frac{a^2\left(1+\beta_{-M}-\beta_M\right)}{b(B_M+1)^3}\left\{\frac{\partial \beta_{M1}}{\partial \varepsilon}+\frac{\partial \beta_{M2}}{\partial \varepsilon}\right\}. \quad \text{(A-3)}$$

Since $\frac{1}{2} > \beta_M = \frac{bK_M}{bK_M+1}$ from (3), $1+\beta_{-M}-\beta_M > 0$. By construction,

$$\frac{\partial \beta_{M1}}{\partial \varepsilon}+\frac{\partial \beta_{M2}}{\partial \varepsilon} = \frac{b}{(1+x_1)^2(1-x_2)^2}\left(2-\beta_1-\beta_2\right)\left(\beta_2-\beta_1-2\varepsilon b(1-\beta_1)(1-\beta_2)\right) \cdot \quad \text{(A-4)}$$

At $\varepsilon_S = \frac{\beta_2-\beta_1}{2b(1-\beta_1)(1-\beta_2)}$, $\frac{\partial \beta_{M1}}{\partial \varepsilon}+\frac{\partial \beta_{M2}}{\partial \varepsilon}=0$ and, for $-K_1 < \varepsilon < \varepsilon_S$, $\frac{\partial \beta_{M1}}{\partial \varepsilon}+\frac{\partial \beta_{M2}}{\partial \varepsilon} > 0$. Thus, A≥0.

The second term in (A-2) is the indirect effect of capital reallocation on overall costs.

$$B: \ \left\{\frac{\partial(q_1^2)}{\partial \varepsilon}\left(\frac{1}{2K_1}\right)+\frac{\partial(q_2^2)}{\partial \varepsilon}\left(\frac{1}{2K_2}\right)\right\} = \frac{a^2}{b(B_M+1)^3}\left\{\begin{array}{l}\frac{\partial \beta_{M1}}{\partial \varepsilon}\left[(\beta_{-M}+1)(1-\beta_{M1})+\beta_{M2}(\beta_{M2}-\beta_{M1})\right] \\ +\frac{\partial \beta_{M2}}{\partial \varepsilon}\left[(\beta_{-M}+1)(1-\beta_{M2})-\beta_{M1}(\beta_{M2}-\beta_{M1})\right]\end{array}\right\}. \quad \text{(A-5)}$$

At ε_S, this effect is zero. For $-K_1 < \varepsilon < \varepsilon_S$, B is positive since

$$\left|\frac{\partial \beta_{M1}}{\partial \varepsilon}\right| > \left|\frac{\partial \beta_{M2}}{\partial \varepsilon}\right| \Leftrightarrow (2-\beta_1-\beta_2)\left\{\beta_2-\beta_1-2\varepsilon b(1-\beta_2)(1-\beta_1)\right\} > 0 \quad \text{(A-6)}$$

and $(\beta_{-M}+1)(1-\beta_{M1})+\beta_{M2}(\beta_{M2}-\beta_{M1}) > (\beta_{-M}+1)(1-\beta_{M2})-\beta_{M1}(\beta_{M2}-\beta_{M1})$. That is, increasing symmetry in the capital distribution of the firms increases the firms' production costs indirectly.

The third term is the direct effect of capital allocation on the overall costs:

$$C: \ \left\{q_1{}^2\left(\frac{\partial\left(\frac{1}{2K_1}\right)}{\partial \varepsilon}\right)+q_2{}^2\left(\frac{\partial\left(\frac{1}{2K_2}\right)}{\partial \varepsilon}\right)\right\} = \frac{a^2}{2b(B_M+1)^2}\left(\frac{\partial \beta_{M1}}{\partial \varepsilon}+\frac{\partial \beta_{M2}}{\partial \varepsilon}\right) \quad \text{(A-7)}$$

This term is zero at ε_S and negative for $-K_1 < \varepsilon < \varepsilon_S$. That is, increasing symmetry in the capital distribution of the firms decreases the production costs directly.

The symmetric solution ε_S is a candidate for solving optimal capital reallocation problem. Plugging (A-3), (A-5), and (A-7) into (A-2), we get

$$\frac{\partial \pi_M}{\partial \varepsilon} = \frac{a^2}{2b(B_M+1)^3} \left\{ \begin{array}{l} \frac{\partial \beta_{M1}}{\partial \varepsilon}\left[y - \beta_M + 2\beta_{M1} y - 2\beta_{M2}(\beta_{M2} - \beta_{M1})\right] \\ + \frac{\partial \beta_{M2}}{\partial \varepsilon}\left[y - \beta_M + 2\beta_{M2} y + 2\beta_{M1}(\beta_{M2} - \beta_{M1})\right] \end{array} \right\} = 0, \tag{A-8}$$

where $y \equiv \beta_{-M} + 1$. For any level of $\beta_1, \beta_2, \beta_{-M}, \varepsilon$ and b, there are three roots satisfying this equation. One of them is $\varepsilon^* = \varepsilon_S = \frac{\beta_2 - \beta_1}{2b(1-\beta_1)(1-\beta_2)} \geq 0$ and the other two are $\varepsilon_S \pm \delta$, where

$$\delta \equiv \frac{\sqrt{R}}{2b(1-\beta_1)(1-\beta_2)\left\{(6+2\beta_{-M})(1-\beta_1)(1-\beta_2) + (1+3\beta_{-M})(2-\beta_1-\beta_2)\right\}}, \quad R \equiv \left\{ \begin{array}{l} (3\beta_{-M} - 1 + 2\beta_S)(2-\beta_1-\beta_2)^3 \beta_S \\ \times\left[(6+2\beta_{-M})(1-\beta_1)(1-\beta_2) + (2-\beta_1-\beta_2)(1+3\beta_{-M})\right] \end{array} \right\}$$

and $\beta_S \equiv \frac{(\beta_1+\beta_2-2\beta_1\beta_2)}{(2-\beta_1-\beta_2)}$. If $R > 0$, δ is a real number. $R > 0$ if and only if

$$\beta_S > \frac{(1-3\beta_{-M})}{2}, \tag{A-9}$$

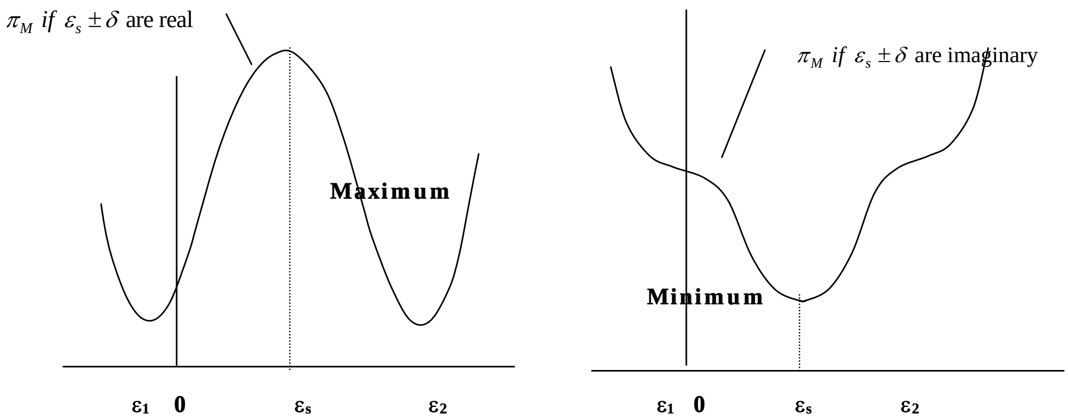
and $R \leq 0$ when $0 < \beta_S < \frac{(1-3\beta_{-M})}{2}$.

Evaluating the second-order condition, $\frac{\partial^2 \pi_M}{\partial \varepsilon^2}$, at $\varepsilon = \varepsilon_S$, we get

$$\begin{aligned} \left.\frac{\partial^2 \pi_M}{\partial \varepsilon^2}\right|_{\varepsilon=\varepsilon_s} &= \frac{16a^2 b}{(B_M+1)^3} \frac{(1-\beta_1)^2(1-\beta_2)^2}{(2-\beta_1-\beta_2)^5}\left\{(1-3\beta_{-M})(2-\beta_1-\beta_2) - 2(\beta_1+\beta_2-2\beta_1\beta_2)\right\} \\ \Leftrightarrow \left.\frac{\partial^2 \pi_M}{\partial \varepsilon^2}\right|_{\varepsilon=\varepsilon_s} &< 0 \quad iff \quad \beta_S > \frac{(1-3\beta_{-M})}{2}. \end{aligned} \tag{A-10}$$

The parameter range that generates real roots for the solutions to the optimal capital reallocation problem coincides with the parameter range that gives $\left.\frac{\partial^2 \pi_M}{\partial \varepsilon^2}\right|_{\varepsilon=\varepsilon_s} < 0$.

Figure 1: The existence of an interior solution to the optimal capital reallocation problem



Case 1: $\left.\frac{\partial^2 \pi_M}{\partial \varepsilon^2}\right|_{\varepsilon=\varepsilon_s} < 0$ Case 2: $\left.\frac{\partial^2 \pi_M}{\partial \varepsilon^2}\right|_{\varepsilon=\varepsilon_s} > 0$

If $\beta_S > \frac{(1-3\beta_{-M})}{2}$, the two roots ($\varepsilon_1 = \varepsilon_S + \delta, \varepsilon_2 = \varepsilon_S - \delta$) are real, and we are in Case 1, in which $\left.\frac{\partial^2 \pi_M}{\partial \varepsilon^2}\right|_{\varepsilon=\varepsilon_s} < 0$, as shown in Figure 1. In this case, the maximum profit occurs at ε_S; that is, the symmetric reallocation is the unique interior solution to the optimal capital reallocation problem. On the other hand, if $0 < \beta_S < \frac{(1-3\beta_{-M})}{2}$, the two roots are imaginary, $\left.\frac{\partial^2 \pi_M}{\partial \varepsilon^2}\right|_{\varepsilon=\varepsilon_s} < 0$, and ε_S yields the global minimum profit. In this case, there is no interior solution to the capital reallocation problem in the range $-K_1 < \varepsilon \leq \frac{K_2 - K_1}{2}$. ■

A2. PROOF OF PROPOSITION 1

From Lemma 1, if $0 < \beta_S < \frac{(1-3\beta_{-M})}{2}$, there is no interior solution to the capital reallocation problem that maintains the M-form. Then, for any merged firm operating under the M-form, it has to be true that $\beta_S > \frac{(1-3\beta_{-M})}{2}$. When $\beta_S > \frac{(1-3\beta_{-M})}{2}$, the optimal capital reallocation occurs at ε_S. Therefore, under the M-form, the optimal capital reallocation of the merged firm is at $\varepsilon_S = \frac{K_2 - K_1}{2}$. ■

A3. PROOF OF LEMMA 2

Let π_{CI} be the merged firm's profit under complete integration. The headquarters' problem is to choose q_i and K_i for each plant *i* that maximizes

$$\begin{aligned}\pi_{CI} &= (a-bQ)q_1 - \frac{q_1^2}{2K_1} + (a-bQ)q_2 - \frac{q_2^2}{2K_2} \\ &= (a-bQ)q_1 + (a-bQ)q_2 - \frac{K_1}{2}\left(\frac{q_1}{K_1}\right)^2 - \frac{K_2}{2}\left(\frac{q_2}{K_2}\right)^2\end{aligned} \tag{A-11}$$

where q_i and K_i are the quantity and the level of capital of each plant *i*, *i*=1,2. When the multi-plant firm operates both plants efficiently, the marginal cost of the firm *mc* must satisfy

$$mc = \frac{q_1}{K_1} = \frac{q_2}{K_2} = \frac{q_1+q_2}{K_1+K_2}. \tag{A-12}$$

Plugging (A-12) into the firm's profit function (A-11), we obtain

$$\begin{aligned}\pi_{CI} &= (a-bQ)(q_1+q_2) - \frac{K_1}{2}\left(\frac{q_1+q_2}{K_1+K_2}\right)^2 - \frac{K_2}{2}\left(\frac{q_1+q_2}{K_1+K_2}\right)^2 \\ &= (a-bQ)(q_1+q_2) - \left(\frac{q_1+q_2}{K_1+K_2}\right)^2\left(\frac{K_1+K_2}{2}\right) \\ &= (a-bQ)q_{CI} - \frac{q_{CI}^2}{2K_M}\end{aligned} \tag{A-13}$$

where $K_M = K_1 + K_2$ and $q_{CI} = q_1 + q_2$. That is, the firm's problem is essentially to choose q_{CI} for a given K_M. Therefore, for a fixed K_M, any capital reallocation results in the same profits π_{CI}. ■

A4. PROOF OF PROPOSITION 2

If $\beta_S > \frac{(1-3\beta_{-M})}{2}$, the highest profit under the M-form occurs at $\varepsilon^* = \frac{K_2-K_1}{2}$, and the profit is $\pi_M^* = \frac{a^2}{2b}\left(\frac{2\beta_S+2\beta_S^2}{(2\beta_S+\beta_{-M}+1)^2}\right)$ from (19). Under complete integration, the merged firm earns $\pi_{CI} = \frac{a^2}{2b}\left(\frac{\beta_{CI}+\beta_{CI}^2}{(\beta_{CI}+\beta_{-M}+1)^2}\right)$ from (15). The headquarters chooses the M-form over complete integration only if $\pi_M^* > \pi_{CI} \Leftrightarrow \frac{(\beta_{-M}+1)(3\beta_{-M}-1)}{(\beta_{-M}-1)^2} > \beta_{CI}$. Combining this with the condition in (17), we get three parameter ranges to consider:

(i) $\frac{(\beta_{-M}+1)(3\beta_{-M}-1)}{(\beta_{-M}-1)^2} > \beta_{CI}$, $\beta_S > \frac{(1-3\beta_{-M})}{2}$.

In this range, $\left.\frac{\partial^2 \pi_M}{\partial \varepsilon^2}\right|_{\varepsilon=\varepsilon_s} < 0$ and $\pi_M^* > \pi_1 + \pi_2$ since $\beta_S > \frac{(1-3\beta_{-M})}{2}$. Also, $\pi_M^* > \pi_{CI}$ since $\frac{(\beta_{-M}+1)(3\beta_{-M}-1)}{(\beta_{-M}-1)^2} > \beta_{CI}$. Therefore, the M-form is optimal for the merged firm in this range.

(ii) $\beta_{CI} > \frac{(\beta_{-M}+1)(3\beta_{-M}-1)}{(\beta_{-M}-1)^2}$, $\beta_S > \frac{(1-3\beta_{-M})}{2}$.

In this range, $\pi_M^* > \pi_1 + \pi_2$ since $\beta_S > \frac{(1-3\beta_{-M})}{2}$. However, the profit under complete integration is higher than the profit under the M-form, i.e., $\pi_{CI} > \pi_M^* > \pi_1 + \pi_2$. Therefore, the merged firm's optimal organizational form is complete integration. In this case, under the M-form, π_M^* is the local maximum, and the global maximum profit occurs at the corner solution.

(iii) $\beta_{CI} > \frac{(\beta_{-M}+1)(3\beta_{-M}-1)}{(\beta_{-M}-1)^2}$, $\beta_S < \frac{(1-3\beta_{-M})}{2}$.

In this range, there is no interior solution to the capital reallocation problem under the M-form because $\beta_S < \frac{(1-3\beta_{-M})}{2}$. As shown in Figure 1, in this case, ε_S gives the global minimum profit and the other two roots are imaginary. As the distribution ε moves away from ε_S, profit is increasing continuously. Hence, $\pi_{CI} > \pi_1 + \pi_2 > \pi_M^*$. The merged firm's optimal organizational form is complete integration.

The range $\beta_{CI} < \frac{(\beta_{-M}+1)(3\beta_{-M}-1)}{(\beta_{-M}-1)^2}$, $\beta_S < \frac{(1-3\beta_{-M})}{2}$ is infeasible given that $\beta_{CI} > 0, \beta_S > 0$ and $\frac{(\beta_{-M}+1)(3\beta_{-M}-1)}{(\beta_{-M}-1)^2}$ always takes the opposite sign of $\frac{2(1-3\beta_{-M})}{3}$. Overall, from (i), (ii), and (iii), the M-form is optimal for a merger between non-identical firms when $\frac{(\beta_{-M}+1)(3\beta_{-M}-1)}{(\beta_{-M}-1)^2} > \beta_{CI}$. Otherwise, for a profitable merger, complete integration is optimal. ■

A5. PROOF OF PROPOSITION 3

Unprofitable Capital Reallocation under the M-form

For a merger between identical firms, $\beta_1 = \beta_2 = \beta_S$. Under the M-form, from (A-1), $\beta_{M1} = \frac{\beta_S + x}{1+x}$ and $\beta_{M2} = \frac{\beta_S - x}{1-x}$, where $x \equiv \varepsilon b(1-\beta_S)$ and $-K_1 < \varepsilon \le 0$. Then, we get $\beta_M = \frac{2(\beta_S - x^2)}{1-x^2} \le 2\beta_S$ for any ε. That is, if the insiders are initially symmetric, any capital reallocation under the M-form only worsens the merged firm's ability to expand output, as $\beta_M \le 2\beta_S = \beta_1 + \beta_2$. The merged firm incurs losses from reallocating capital under the M-form. The first-order condition is

$$\frac{\partial \pi_M}{\partial \varepsilon} = \frac{a^2}{2b}\left(\frac{1}{(B_M+1)^3}\right)\left\{\begin{array}{l}(1+\beta_{-M})\left(\frac{\partial \beta_{M1}}{\partial \varepsilon} + \frac{\partial \beta_{M2}}{\partial \varepsilon}\right) + 2\beta_{-M}\left(\frac{\partial \beta_{M1}}{\partial \varepsilon}\beta_{M1} + \frac{\partial \beta_{M2}}{\partial \varepsilon}\beta_{M2}\right) \\ + \frac{\partial \beta_{M1}}{\partial \varepsilon}(1+2\beta_{M2})(\beta_{M1}-\beta_{M2}) - \frac{\partial \beta_{M2}}{\partial \varepsilon}(1+2\beta_{M1})(\beta_{M1}-\beta_{M2})\end{array}\right\}. \quad \text{(A-14)}$$

For any ε in the range $-K_1 < \varepsilon < 0$, $(\beta_{M1} - \beta_{M2}) < 0$. Since $\frac{\partial \beta_{M1}}{\partial \varepsilon} + \frac{\partial \beta_{M2}}{\partial \varepsilon} = \frac{(1-\beta_S)^2 b}{(1+x)^2} - \frac{(1-\beta_S)^2 b}{(1-x)^2} < 0$ and $\frac{\partial \beta_{M1}}{\partial \varepsilon}\beta_{M1} + \frac{\partial \beta_{M2}}{\partial \varepsilon}\beta_{M2} = \left(\frac{\partial \beta_{M1}}{\partial \varepsilon} + \frac{\partial \beta_{M2}}{\partial \varepsilon}\right)\beta_{M1} + \frac{\partial \beta_{M2}}{\partial \varepsilon}(\beta_{M2} - \beta_{M1}) < 0$, evaluating (A-14), we get $\left.\frac{\partial \pi_M}{\partial \varepsilon}\right|_{\varepsilon \ne 0} < 0$. Thus, for a merger between two identical firms, the M-form is never optimal. The merger can be profitable only under complete integration.

Profitable Horizontal Merger between Identical Firms

The merged firm earns $\pi_{CI} = \frac{a^2}{2b}\left(\frac{\beta_{CI} + \beta_{CI}^2}{(\beta_{CI} + \beta_{-M} + 1)^2}\right)$ under complete integration. From Lemma 2, capital reallocation is irrelevant if a merged firm operates under complete integration. Let $\beta_{CI} \equiv \frac{bK_M}{bK_M + 1}$ be the parameter that indicates the merged firm's aggressiveness in production under complete integration. By construction, $K_M = \frac{\beta_{CI}}{b(1-\beta_{CI})}$ and $K_1 + K_2 = \frac{2\beta_S}{b(1-\beta_S)}$. Since $K_M = K_1 + K_2$, we get $\frac{\beta_{CI}}{b(1-\beta_{CI})} = \frac{2\beta_S}{b(1-\beta_S)}$. Thus, $\beta_S < \beta_{CI} < 2\beta_S$.

Before the merger, the combined profits of the two firms are $2\pi_i^* = \frac{a^2}{2b}\left(\frac{2\beta_S + 2\beta_S^2}{(2\beta_S + \beta_{-M} + 1)^2}\right)$. The merger is profitable if

$$\pi_{CI} > 2\pi_i^* \Leftrightarrow \frac{a^2}{2b}\left(\frac{\beta_{CI}+\beta_{CI}^2}{(\beta_{CI}+\beta_{-M}+1)^2}\right) > \frac{a^2}{2b}\left(\frac{2\beta_S+2\beta_S^2}{(2\beta_S+\beta_{-M}+1)^2}\right). \quad \text{(A-15)}$$

Since $\beta_S = \frac{\beta_{CI}}{2-\beta_{CI}}$, (A-15) reduces to $\beta_{CI} > \frac{(\beta_{-M}+1)(3\beta_{-M}-1)}{(\beta_{-M}-1)^2}$. Therefore, horizontal mergers between identical firms are profitable under complete integration if $\beta_{CI} > \frac{(\beta_{-M}+1)(3\beta_{-M}-1)}{(\beta_{-M}-1)^2}$. ■

A6. PROOF OF PROPOSITION 4

When the merged firm operates two symmetric divisions under the M-form,

(i) $\beta_M = 2\beta_S = \frac{2(\beta_1+\beta_2-2\beta_1\beta_2)}{(2-\beta_1-\beta_2)} > \beta_1+\beta_2$.

(ii) Thus, $Q_M = \frac{a}{b}\left(\frac{\beta_M+\beta_{-M}}{\beta_M+\beta_{-M}+1}\right) > \frac{a}{b}\left(\frac{\beta_1+\beta_2+\beta_{-M}}{\beta_1+\beta_2+\beta_{-M}+1}\right) = Q^*$ and $P_M = a\left(\frac{1}{\beta_M+\beta_{-M}+1}\right) < a\left(\frac{1}{\beta_1+\beta_2+\beta_{-M}+1}\right) = P^*$, where Q_M, P_M are the market output and the market price after the merger, respectively.

(iii) Consumer welfare increases as a result.

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Supplementary Appendix

Consider the problem of distributing a fixed level of capital K_M across a pair of firms, firm 1 and firm 2, where $K_M = K_1 + K_2$. Assume that $K_1 \leq K_2$. K_M is distributed according to the distribution rule ε in (16). With a linear demand, firm *i*'s profit is

$$\pi_i = (a - bQ)q_i - C(q_i, K_i) \qquad i=1,2,\ldots\ldots N. \qquad \text{(B-1)}$$

Assume that $\frac{\partial C(q_i,K_i)}{\partial q_i} > 0, \frac{\partial^2 C(q_i,K_i)}{\partial q_i^2} \geq 0, \frac{\partial C(q_i,K_i)}{\partial K_i} < 0, \frac{\partial^2 C(q_i,K_i)}{\partial K_i^2} \geq 0,$ and $\frac{\partial^2 C(q_i,K_i)}{\partial q_i \partial K_i} < 0$. The first-order condition for firm *i*'s output decision is given by

$$a - bQ - mc^i(q_i, K_i) - bq_i = 0\,, \qquad \text{(B-2)}$$

where $mc^i(q_i, K_i) \equiv \frac{\partial C(q_i,K_i)}{\partial q_i}$. Differentiating totally (B-2), we get

$$dq_i = -\beta_i dQ + \gamma_i dK_i\,, \qquad \text{(B-3)}$$

where $\beta_i \equiv -\left(\frac{b}{b+mc_q^i}\right)$, $\gamma_i \equiv \left(\frac{-mc_K^i}{b+mc_q^i}\right)$, $mc_q^i \equiv \frac{\partial mc_i(q_i,K_i)}{\partial q_i} \geq 0$, and $mc_K^i \equiv \frac{\partial mc_i(q_i,K_i)}{\partial K_i} < 0$. Then,

$$dQ = \frac{\sum_i \gamma_i dK_i}{B+1} \qquad \text{(B-4)}$$

where $B \equiv \sum_i \beta_i$, i=1,2,.....,N. An additional unit of capital affects firm *i*'s profit by

$$\frac{d\pi_i}{dK_i} = bq_i\left(\frac{(B-\beta_i)\gamma_i}{B+1}\right) - C_K^i \qquad \text{(B-5)}$$

where $C_K^i \equiv \frac{\partial C(q_i,K_i)}{\partial K_i}$. A small amount of capital transfer from firm 2 to firm 1 affects joint profits by

$$\frac{d\pi_1}{dK}+\frac{d\pi_2}{(-dK)}=b\left(\frac{(B-\beta_1)\gamma_1 q_1-(B-\beta_2)\gamma_2 q_2}{B+1}\right)-C_K^1+C_K^2 \tag{B-6}$$

Let K_S be the level of capital for each firm when total capital K_M is evenly distributed across the two firms. At K_S, $q_1=q_2=q_s$, $\beta_1=\beta_2=\beta_s$, $\gamma_1=\gamma_2=\gamma_s$, $C_K^1=C_K^2$, and thus, $\left(\frac{d\pi_1}{dK}+\frac{d\pi_2}{(-dK)}\right)\Big|_{K_S}=0$. Taking the second derivative with respect to K and evaluating it at the symmetric solution K_S, we get

$$\frac{d^2\pi_1}{dK^2}+\frac{d^2\pi_2}{(-dK)^2}\bigg|_{K_S}=\underbrace{2\left(b\left(\frac{dq_i}{dK}-\frac{dQ}{dK}\right)-mc_K^i\right)\bigg|_{K_S}}_{A}\underbrace{-2C_{KK}^i\Big|_{K_S}}_{B}\underbrace{+2bq_i\left(\frac{d^2q_i}{dK^2}-\frac{d^2Q}{dK^2}\right)\bigg|_{K_S}}_{C},\quad i=1,2. \tag{B-7}$$

If $\frac{d^2\pi_1}{dK^2}+\frac{d^2\pi_2}{(-dK)^2}\Big|_{K_S}<0$, increasing symmetry (reducing asymmetry) in the capital distribution of the two firms improves joint profits, and vice versa. The sign of the first term A in (B-7) is always non-negative since $\left(\frac{dq_i}{dK}-\frac{dQ}{dK}\right)\Big|_{K_S}=\gamma_i(K_S)\geq 0$ and $mc_K^i\leq 0$. B is always non-positive if returns to capital are diminishing. The sign of C depends on the shape of the marginal cost function:

$$\left(\frac{d^2q_i}{dK^2}-\frac{d^2Q}{dK^2}\right)\bigg|_{K_S}=\frac{\partial\gamma_i}{dK}\bigg|_{K_S}=\frac{-mc_{KK}^i(b+mc_q^i)+mc_K^i mc_{qK}^i}{(b+mc_q^i)^2}\bigg|_{K_S}$$

When the marginal cost function is strictly convex in output and capital, $mc_{qq}^i>0$ and $mc_{KK}^i>0$.

(i) If marginal cost is constant in output, $mc_q^i=0$, and strictly convex in capital,

$$C_{KK}^i<0,\quad \gamma_i=\frac{-mc_K^i}{b},\quad \text{and}\quad \frac{\partial\gamma_i}{\partial K}=-\frac{mc_{KK}^i}{b}<0$$

Therefore, A>0, B<0, and C<0. If B and C are larger than A, then $\frac{d^2\pi_1}{dK^2}+\frac{d^2\pi_2}{(-dK)^2}\Big|_{K_S}<0$.

(ii) If marginal cost is strictly convex in output and linear in capital, $mc_K^i=\delta$,

$$C_{KK}^i=0,\quad \gamma_i=\frac{-\delta}{b+mc_q^i},\quad \text{and}\quad \frac{\partial\gamma_i}{\partial K}=0.$$

Therefore, A>0, B=C=0, which implies that $\frac{d^2\pi_1}{dK^2}+\frac{d^2\pi_2}{(-dK)^2}\Big|_{K_S}>0$.

(iii) If marginal cost is linear in output and capital, i.e., $mc_q^i=\eta$, $mc_K^i=\delta$,

$$C^i_{KK} = 0, \quad \gamma_i = \tfrac{-\delta}{b+\eta}, \quad \text{and } \tfrac{\partial \gamma_i}{\partial K} = 0$$

Therefore, A>0, B=C=0, which implies that $\frac{d^2\pi_1}{dK^2} + \frac{d^2\pi_2}{(-dK)^2}\Big|_{K_S} > 0$.

(iv) In the current framework,

$$C^i_{KK} = \tfrac{q_i^2}{K_i^3}, \quad \gamma_i = \tfrac{q_i}{K_i(bK_i+1)}, \quad \text{and } \tfrac{\partial \gamma_i}{\partial K} < 0.$$

Therefore, A>0, B<0, and C<0. Increasing symmetry in capital distribution increases joint profits if B and C are larger than A, i.e., $\beta_S > \frac{(1-3\beta_{-M})}{2}$. From (i) through (iv), it is clear that distributing capital symmetrically across firms improves joint profits only if the marginal cost function is strictly convex in capital. The framework of Salant and Shaffer (1998, 1999) belongs to the case (iii) where marginal cost is not strictly convex in capital nor in output, which is the reason that distributing capital asymmetrically across firms improves joint profits in their studies.